MONOTONICITY VIOLATIONS UNDER PLURALITY WITH A RUNOFF: THE CASE OF FRENCH PRESIDENTIAL ELECTIONS

Umut Keskin[a], **M. Remzi Sanver**[b,c], **H. Berkay Tosunlu**[b]

[a] İstanbul Bilgi University. Kazım Karabekir Cad. No: 2/13 34060. Eyüp. İstanbul, TURKEY.

[b] Université Paris-Dauphine, Université PSL, CNRS, LAMSADE, 75016 Paris, FRANCE.

[c] Corresponding author: remzi.sanver@dauphine.fr

Abstract

A voting rule is monotonic if a winning candidate never becomes a loser by being raised in voters' rankings of candidates, ceteris paribus. Plurality with a runoff is known to fail monotonicity. To see how widespread this failure is, we focus on French presidential elections since 1965. We identify mathematical conditions that allow a logically conceivable scenario of vote shifts between candidates that may lead to a monotonicity violation. We show that eight among the ten elections held since 1965 (those in 1965 and 1974 being the exceptions) exhibit this theoretical vulnerability. To be sure, the conceived scenario of vote shifts that enables a monotonicity violation may not be plausible under the political context of the considered election. Thus, we analyze the political landscape of these eight elections and argue that for two of them (2002 and 2007 elections), the monotonicity violation scenario was plausible within the conjuncture of the time.

Keywords: French presidential elections, plurality with a runoff, monotonicity.

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1. Introduction

A voting rule is monotonic if a winning candidate never becomes a loser by being raised in voters' rankings of candidates, leaving everything else unchanged. This is a simple and weak condition that is known for a long time.[1] As weak as it may be, it is not satisfied by every popular voting rule. For example, as shown by Smith (1973), all scoring elimination rules fail monotonicity.[2]

When a voting rule fails monotonicity, a candidate may lose an election she would have won because of an additional support he gets. This casts ambiguity on whether a candidate should try to convince voters on the rightness of the cause she defends. Such a perverse incentive contradicts a basic principle of democracy that connects the value of democratic outcomes to the virtues of deliberation.[3]

A very famous member of the family of scoring elimination rules is plurality with a runoff (PwR) which is used by many countries to elect by popular vote the president or mayors or parliament members in single constituencies.[4] Given that PwR, in spite of its non-monotonicity, is used in political elections, it is worth asking the frequency of observing this perverse behavior. Lepelley et al. (1996) derive analytical representations for the proportion of voting situations where PwR fails monotonicity, when there are three candidates only. As an analysis in the same direction, the frequency of monotonicity failures under Instant Runoff Voting (IRV) is estimated by Ornstein and Norman (2014) in a spatial election model and by Miller (2017) through simulated data sets, again in elections with three candidates. Their findings are relevant for PwR which is equivalent to IRV when there are only three candidates.

We pose a question of the same spirit with a different approach. We look at real election data and observe the frequency of elections that are susceptible to monotonicity violations. There is a literature that analyzes monotonicity violations under IRV over real election data such as Bradley (1995), Miller (2017), Ornstein and Norman (2014), Graham-Squire and Zayatz (2020). As to the best of our knowledge, our analysis is the first to do this for PwR. We focus on a well-known and important application of PwR: the French presidential elections since 1965. From this date to our day, we analyze each election, held under PwR, to conclude whether that election is susceptible to exhibit a monotonicity violation or not.

Our analysis needs to handle a subtlety: The literature of social choice theory typically conceives PwR as a voting rule that determines the winner based on voters' rankings of candidates (see e.g. Keskin et al. (2020)). On the other hand, PwR is implemented in France as a protocol with two successive rounds where voters are allowed to vote for at most one candidate at every round. Thus, our analysis must adapt the concepts of the literature to this

[1] Black (1958) and Brams and Fishburn (2002) give a comprehensive account of this condition, which should be distinguished from other stronger monotonicity conditions that are more oriented to the implementability of collective choice rules. For those different monotonicities of implementation theory, once can see Maskin and Sjöström (2002).

[2] Felsenthal and Nurmi (2017) present a detailed account of monotonicity failures.

[3] See Doron and Kronick (1977) for arguments against using non-monotonic voting rules.

[4] Golder (2005) discusses the use of PwR in different elections around the world.

(restricted) informational basis.[5] Moreover, as voters' rankings of candidates is not available information, theoretical conclusions about the outcome of the second round are limited. To handle this issue, we consider a stronger version of monotonicity that overlooks the outcome of the second round and identify conditions that render a voting situation vulnerable to a change in the candidates who go for a runoff (instead of a change ın the winning candidate), when the winning candidate receives more votes.

In a voting situation that does not exhibit a failure of this stronger monotonicity, the failure of monotonicity is logically impossible. On the other hand, whether the failure of strong monotonicity implies a failure of monotonicity depends on the outcome of the runoff, which is now between a different pair of candidates. In fact, a voting situation that violates strong monotonicity may satisfy monotonicity because the winner may prevail under the new pair of candidates that go to a runoff. To address this issue, we consider the political context of the election under consideration by referring to pre-election opinion polls as well as papers that analyze French presidential elections. In a similar vein, we discuss the plausibility of the overall political scenario implied by the monotonicity violation in question.

We consider every French presidential election held under PwR, from the first one in 1965 to our day. There are 10 of them: 1965, 1969, 1974, 1981, 1988, 1995, 2002, 2007, 2012 and 2017. It turns out that among the ten French presidential elections held under PwR, only two of them, namely those in 1965 and 1974, satisfy strong monotonicity which render these elections logically immune to a monotonicity violation. On the other hand, the remaining eight elections fail strong monotonicity, thus admitting a logical possibility of exhibiting monotonicity violation. Discussing the political context of these eight elections, we argue that in six of them (namely those in 1969, 1981, 1988, 1995, 2012 and 2017) the logical possibility of changing the pair of candidates that go for a runoff would not effectively change the election outcome. Our analysis leads to the conclusion that in two of the ten French presidential elections (namely those in 2002 and 2007), the scenario that leads to a monotonicity violation was plausible under the political landscape of the time, making a monotonicity violation practically possible.

Section 2 presents the basic notions within the standard conceptual framework of social choice theory. Section 3 adapts these notions to the informational basis of the French presidential elections and gives our theoretical results on the logical possibility that monotonicity violations occur. Section 4 presents election results and identifies the elections that admit a logical possibility of exhibiting monotonicity violation. Section 5 discusses these findings with respect to the political context of the election in question. Section 6 makes some final remarks.

2. Basic notions

We consider a set of *voters* who will elect one person from a set of *candidates*. Each voter has a strict ranking over candidates to which we refer as this voter's *preference*. A *preference profile* is the list of all voters' preferences.

Example 1: In an election with 93 voters and three candidates (called C_1, C_2, and C_3), we write

[5] The qualification "restricted" implicitly assumes that voters' preferences do not change between the two rounds and that strategic voting is not a concern. See Conitzer and Sandholm (2005) for a formalization of PwR as a protocol.

P		
42	27	24
C_1	C_2	C_3
C_2	C_3	C_1
C_3	C_1	C_2

for the preference profile *P* where 42 voters rank C_1 as first, C_2 as second, C_3 as last; 27 voters rank C_2 as first, C_3 as second, C_1 as last; 24 voters rank C_3 as first, C_1 as second, C_2 as last.

A *voting rule* elects a single candidate from every preference profile. The *plurality score* of a candidate at a given preference profile is the number of voters who top rank that candidate. So, at the preference profile *P* of Example 1, the plurality score of C_1, which we denote by $\#C_1$, is 42. Similarly, $\#C_2$ = 27 and $\#C_3$ = 24. Under *plurality with a runoff* (PwR), if the plurality winner receives a majority of the votes then the election concludes in one round. If not, then the plurality winner and the plurality second best go for a runoff whose winner is the candidate who receives a majority support against the other according to the preference profile under consideration. So, at the preference profile *P*, at the runoff, C_1 gets 66 votes against C_2 who gets 27 votes, which makes C_1 the candidate elected by PwR.

Preference profiles that admit ties are extremely rare in presidential elections held under popular vote. As a result, we only consider preference profiles with no ties.

We now define our core condition:

MONOTONICITY (WITH RESPECT TO RANKINGS): Lifting an elected candidate *C* in one or more voters' rankings keeping all other candidates' ranks the same (except for those affected by the lifting of *C*) should not prevent *C* from being elected.

Example 2: To see that PwR violates monotonicity, we compare below the preference profile *P* with another preference profile *Q*:

P				*Q*		
42	27	24		46	23	24
C_1	C_2	C_3		C_1	C_2	C_3
C_2	C_3	C_1		C_2	C_3	C_1
C_3	C_1	C_2		C_3	C_1	C_2

Note that *Q* is obtained from *P* through the lifting of C_1 to the top rank by 4 of the C_2 C_3 C_1type voters (in the middle column). At *Q*, the plurality winner C_1 and the plurality second best, which is now C_3, go to a runoff where C_3 gets 47 votes against C_1 who gets 46 votes, which makes C_3 the candidate elected by PwR. Thus, C_1 who is the PwR winner at *P* is lifted in 4 voters' preferences to the top everything else being the same. This lifting results in the preference profile Q where C_1 now loses under PwR.

We refer to this kind of monotonicity violation as *upward violation* to distinguish it from *downward violation* where a candidate C who loses at some preference profile becomes the winner when one or more voters lower C in their rankings, everything else being the same. Clearly, the two types are equivalent in the sense that if there is an upward violation from one preference profile to the other then there will be a downward violation when the order of these two preference profiles is reversed. For instance, in the above example, there is a downward violation going from Q to P where a loser (candidate C_1) at Q, by being lowered in 4 voters' preferences becomes the winner at P. Nevertheless, we need this conceptual distinction because while considering French presidentials, we will have as reference a single preference profile, which is the one that occurred during the considered election.

3. Theoretical results

In French presidential elections, the preference profile is not an available information and PwR is implemented in two successive rounds where voters are asked to vote for a single candidate at every round. We adapt our conceptual framework to this fact, by considering PwR as a protocol that determines the winning candidate based on the number of votes that each candidate receives in the first round, as well as the votes that the two competitors receive at the runoff.[6] To formalize this, let there be $K > 2$ candidates. We assume, without loss of generality, $\sum_1^K \#C_j = 100$ with $\#C_j$ being a non-negative real number for each $j = 1,\ldots, K$. We also let $\#C_1 > \#C_2 > \cdots > \#C_K$, without loss of generality. So $C_1$and C_2 go for a runoff (unless $\#C_1 > 50$ in which case C_1 is the winner) and the one who receives more votes than the other in the second round becomes the PwR winner.

Monotonicity can be redefined in this setting:

MONOTONICITY (WITH RESPECT TO VOTES): Transferring votes in the first round to the winning candidate C keeping all other candidates' votes the same (except for those affected by the vote transfer to C) should not prevent C from being elected.

We consider the violation of the following stronger version of monotonicity:

STRONG MONOTONICITY (WITH RESPECT TO VOTES): Transferring votes in the first round to the winning candidate C keeping all other candidates' votes the same (except for those affected by the vote transfer to C) should not change the candidates who go for a runoff.

Again, we distinguish between upward and downward violations:

3.1 Upward violations

An upward violation of strong monotonicity occurs when a vote shift to the PwR winner from one or more candidates, ceteris paribus, results in a change in the candidates who go for a runoff.

[6] Under this adaptation that overlooks preferences, the question on whether voters' preferences change between the two rounds vanish. In a similar vein, strategic voting is not a concern. See Footnote 5.

This may happen in different ways, which we classify as follows.

Upward violation by the plurality winner (PW-upward): A PW-upward violation is an upward violation that occurs when the plurality winner (denoted C_1) is the PwR winner.

We make sub-classifications of PW-upward violations depending on the candidate(s) from whom C_1 receives votes.

PW-upward(2): C_1 is the winner and by receiving votes from C_2, ceteris paribus, goes to the runoff with C_3 (instead of C_2) without exceeding the threshold of 50 votes.[7]

Example 3 (PW-upward(2)): We adapt Example 2 in Section 2 to the setting of this section. There are three candidates: $C_1, C_2,$ and C_3. The tables below give two possible voting situations in the first round of an election. We assume that at the second round C_1 beats C_2. The voting situation at the left (S1) is the initial situation; the one at the right (S2) is obtained from S1 by transferring 4 votes from C_2 to C_1. The runoff pair at S1 is C_1, C_2 while at S2 is C_1, C_3.

S1

Candidate	Vote
C_1	42
C_2	27
C_3	24

S2

Candidate	Vote
C_1	46
C_2	23
C_3	24

Proposition 1: A PW-upward(2) violation occurs if and only if $50 - \#C_1 > \#C_2 - \#C_3$.

Proof: For the necessity, let a PW-upward(2) violation occur. So by transferring x votes from C_2 to C_1, ceteris paribus, C_1 goes to the runoff with C_3. This implies $\#C_2 - x < \#C_3$. Moreover, to ensure that C_1 does not exceed the threshold of 50 votes after the transfer, we must have $\#C_1 + x < 50$. Combining these two inequalities, we get $50 - \#C_1 > \#C_2 - \#C_3$.

For the sufficiency, let $50 - \#C_1 > \#C_2 - \#C_3$. Take some x that satisfies $50 - \#C_1 > x > \#C_2 - \#C_3$. Clearly, by transferring x votes from C_2 to C_1, ceteris paribus, C_1 goes to the runoff with C_3 without exceeding the threshold of 50 votes, hence showing a PW-upward(2) violation. Q.E.D.

The concept of a PW-upward(2) violation can be generalized for any k with $K - 1 \geq k \geq 2$ as follows:

PW-upward(k): C_1 is the winner and by receiving votes from C_2,…, C_k, ceteris paribus, goes to the runoff with C_{k+1} (instead of C_2) without exceeding the threshold of 50 votes.

Example 4 (PW-upward(3)): There are 4 candidates: C_1, C_2, C_3 and C_4. The tables below give two possible voting situations in the first round of an election. We assume that at the second round C_1 beats C_2. The voting situation at the left (S1) is the initial situation; the one at the right

[7] Although we postpone the discussion on the runoff outcome to the next section where we use real election data, we note right away that the condition "C_1 not exceeding the threshold of 50 votes" ensures that C_1 does not become the PwR winner without needing a runoff.

(S2) is obtained from S1 by transferring to C_1, 4 votes from C_2 and 3 votes from C_3. The runoff pair at S1 is C_1, C_2 while at S2 is C_1, C_4.

S1			**S2**	
Candidate	**Vote**		**Candidate**	**Vote**
C_1	30		C_1	37
C_2	25		C_2	21
C_3	23		C_3	20
C_4	22		C_4	22

Proposition 2: For $K-1 \geq k \geq 2$, a PW-upward(k) violation occurs if and only if $50 - \#C_1 > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$.

Proof: For the necessity, let a PW-upward(k) violation occur for some for some k with $K - 1 \geq k \geq 2$. So by transfering x_i votes from C_i to C_1 for each *i* = 2,…, k, ceteris paribus, C_1 goes to the runoff with C_{k+1}. This implies $\#C_i - x_i < \#C_{k+1}$ for each *i* = 2,…, k, establishing $\sum_{i=2}^{k} x_i > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$. Moreover, to ensure that C_1 does not exceed the threshold of 50 votes after the transfer, we must have $\#C_1 + \sum_{i=2}^{k} x_i < 50$. Combining these two inequalities, we get $50 - \#C_1 > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$.

For the sufficiency, let $50 - \#C_1 > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$. For each *i* = 2,…, k, pick some vote transfer $x_i > C_i - C_{k+1}$. Since $50 - \#C_1 > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$., one can choose each x_i sufficiently close to $C_i - C_{k+1}$ so that $50 - \#C_1 > \sum_{i=2}^{k} x_i > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$, which ensures that by transferring x_i votes from each *i* = 2,…, k to C_1, ceteris paribus, C_1 goes to the runoff with C_{k+1} without exceeding the threshold of 50 votes, hence showing a PW-upward(k) violation. Q.E.D.

The next result helps to limit the number of PW-upward violation subcases we need to consider on a given election data:

Proposition 3: For $K - 2 \geq k \geq 2$, if a PW-upward(k) violation does not occur, then a PW-upward(k+1) violation does not occur either.

Proof: Assume a PW-upward(k) violation does not occur for some k = 2,…, *K*-2. By Proposition 2, we have $50 - \#C_1 \leq \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$. As $\#C_{k+1} > \#C_{k+2}$ by definition, $\sum_{i=2}^{k+1}(\#C_i - \#C_{k+2}) > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$, implying $50 - \#C_1 \leq \sum_{i=2}^{k+1}(\#C_i - \#C_{k+2})$, which, again by Proposition 2, shows that a PW-upward(k+1) violation does not occur. Q.E.D.

We now consider upward violations by a plurality loser to which we refer as PL-upward violations. This general class refers to situations in which an upward violation occurs when C_2 is the PwR winner. As in PW-upward violations, this family comes with possible sub-classes with different scenarios. We present them below.

PL-upward(1): C_2 is the winner and by receiving votes from C_1, ceteris paribus, goes to runoff with C_3 (instead of C_1) without exceeding the threshold of 50 votes.

Example 5 (PL-upward(1)): There are three candidates: C_1, C_2, and C_3. The tables below give two possible voting situations in the first round of an election. We assume that at the second round C_2 beats C_1. The voting situation at the left (S1) is the initial situation; the one at the right (S2) is obtained from S1 by transferring 6 votes from C_1 to C_2. The runoff pair at S1 is C_1, C_2 while at S2 is C_2, C_3.

S1

Candidate	**Vote**
C_1	36
C_2	33
C_3	31

S2

Candidate	**Vote**
C_1	30
C_2	39
C_3	31

Proposition 4: A PL-upward(1) violation occurs if and only if $50 - \#C_2 > \#C_1 - \#C_3$.

Proof: For the necessity, let a PL-upward(1) violation occur. So by transferring x votes from C_1 to C_2, ceteris paribus, C_2 goes to the runoff with C_3. This implies $\#C_1 - x < \#C_3$. Moreover, to ensure that C_2 does not exceed the threshold of 50 votes after the transfer, we must have $\#C_2 + x < 50$. Combining these two inequalities, we get $50 - \#C_2 > \#C_1 - \#C_3$.

For the sufficiency, let $50 - \#C_2 > \#C_1 - \#C_3$. Take some x that satisfies $50 - \#C_2 > x > \#C_1 - \#C_3$. Clearly, by transferring x votes from C_1 to C_2, ceteris paribus, C_2 goes to the runoff with C_3 without exceeding the threshold of 50 votes, hence showing a PL-upward(1) violation. Q.E.D.

We also define and analyze higher order PL-upward violations where C_2 is lifted by receiving votes from more than one candidate (up to C_k, with k$= 3, \dots, K-1$).

PL-upward(k): C_2 is the winner and by receiving votes from C_1, C_3 …, C_k, ceteris paribus, goes to the runoff with C_{k+1} (instead of C_1) without exceeding the threshold of 50 votes.

Example 6 (PL-upward(3)): There are 4 candidates: C_1, C_2, C_3 and C_4. The tables below give two possible voting situations in the first round of an election. We assume that at the second round C_2 beats C_1. The voting situation at the left (S1) is the initial situation; the one at the right (S2) is obtained from S1 by transferring to C_2, 6 votes from C_1 and 3 votes from C_3. The runoff pair at S1 is C_1, C_2 while at S2 is C_2, C_4.

S1

Candidate	**Vote**
C_1	28
C_2	25
C_3	24
C_4	23

S2

Candidate	**Vote**
C_1	22
C_2	34
C_3	21
C_4	23

Proposition 5: For k$= 3, \dots, K-1$, a PL-upward(k) violation occurs if and only if $50 - \#C_2 > (\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$.

Proof: For the necessity, let a PL-upward(k) violation occur for some for some k with $K - 1 \geq k \geq 3$. So by transfering x_i votes from C_i to C_2 for each *i* = 1,3, …, k, ceteris paribus, C_2 goes to the runoff with C_{k+1}. This implies $\#C_i - x_i < \#C_{k+1}$ for each for each i = 1,3, …, k, establishing $x_1 + \sum_{i=3}^{k} x_i > (\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$. Moreover, to ensure that C_2 doesn't exceed the threshold of 50 votes after the transfer, we must have $\#C_2 + x_1 + \sum_{i=3}^{k} x_i < 50$. Combining these two inequalities, we get 50 – $\#C_2 >$ $(\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$.

For the sufficiency, let 50 – $\#C_2 > (\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$. For each *i* = 3,…,k, pick some vote transfer x_i > $C_i - C_{k+1}$. Since 50 – $\#C_2 >$ $(\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$, one can choose each x_i sufficiently close to$\#C_i - \#C_{k+1}$ so that 50 – $\#C_1 > x_1 + \sum_{i=3}^{k} x_i > \sum_{i=3}^{k}(\#C_i - \#C_{k+1})$, which ensures that by transfering x_i votes from each *i* = 1, 3, …, k to C_2, ceteris paribus, C_2 goes to the runoff with C_{k+1} without exceeding the threshold of 50 votes, hence showing a PL-upward(k) violation. Q.E.D.

Proposition 6:

(i) If a PL-upward(1) violation does not occur, then a PL-upward(3) violation does not occur either.

(ii) For $K - 2 \geq k \geq 3$, if a PL-upward(k) violation does not occur, then a PL-upward(k+1) violation does not occur either.

Proof: We only prove (ii), as the proof of (i) goes mutatis mutandis.

Assume a PL-upward(k) violation does not occur for some k = 3,…, *K*-2. By Proposition 5, we have 50 – $\#C_2 \leq (\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$.

As $\#C_{k+1}$> $\#C_{k+2}$ by definition, we have $(\#C_1 - \#C_{k+2}) + \sum_{j=3}^{k+1}(\#C_j - \#C_{k+2})$ > $(\#C_1 - \#C_{k+1}) + \sum_{j=3}^{k}(\#C_j - \#C_{k+1})$, implying 50 – $\#C_2 \leq (\#C_1 - \#C_{k+2}) + \sum_{j=3}^{k+1}(\#C_j - \#C_{k+2})$ which, again by Proposition 5, shows that a PL-upward(k+1) violation does not occur. Q.E.D.

3.2 Downward violations

A downward violation of strong monotonicity occurs when a vote shift from a PwR loser C to one of her competitors (everything else being the same) results in a change in the candidates who go for a runoff. This change can result in C becoming the PwR winner (hence entail a failure of monotonicity) only if C remains in the runoff after the vote transfer. Clearly, this necessitates that C is initially in the runoff as well. Moreover, we know from Theorem 3.1 and Proposition 5.2 in Keskin et al. (2020) that C must initially be the plurality winner. We define downward violation of strong monotonicity accordingly.

Downward violation: The plurality winner C_1 who is a PwR loser, by losing votes to some candidate keeping all other candidates' votes the same (except for the one affected by the vote transfer from C_1), goes to the runoff with a different candidate than his original competitor.

Note that the condition of not exceeding the threshold of 50 votes used for upward violations is vacuous for downward violations.

A downward violation entails a scenario where votes of C_1 are transferred to C_k for some $k = 3, \dots, K$ to ensure a runoff between C_1 and C_k. Accordingly, we specify such a violation as downward(k).

Example 7 (Downward(3)): There are 4 candidates: C_1, C_2, C_3 and C_4. The tables below give two possible voting situations in the first round of an election. We assume that at the second round C_2 beats C_1. The voting situation at the left (S1) is the initial situation; the one at the right (S2) is obtained from S1 by C_1 losing 2 votes to C_3. The runoff pair at S1 is C_1, C_2 while at S2 is C_1, C_3.

S1			**S2**	
Candidate	**Vote**		**Candidate**	**Vote**
C_1	31		C_1	29
C_2	27		C_2	27
C_3	26		C_3	28
C_4	16		C_4	16

The following characterization follows from Proposition 5.3 in Keskin et al. (2020).

Proposition 7: Given any $k = 3, \dots, K$, a downward(k) violation occurs if and only if C_2 is the PwR winner and $\#C_1 - \#C_2 > \#C_2 - \#C_k$.

The following corollary to Proposition 7 limits the number of cases we consider for downward monotonicity violations.

Corollary 1: Given any $k = 3, \dots, K - 1$, if a downward(k) violation does not occur, then a downward(k+1) violation does not occur as well.

4. French presidentials: Logically possible monotonicity violations

For each of the ten presidential elections that took place in France between 1965 and 2017 under PwR, we give a table that contains

- the full list of candidates together with the political parties to which they belong[8];
- the vote percentage that every candidate receives in the first round as well as in the second round for those two candidates that go for a runoff. (The plurality winners of the first round are written in bold under the "CANDIDATE" column, and the PwR winners are pointed out by their boldfaced round 2 votes in the last column.)

[8] Although in French politics candidates typically stand out more than the party they are associated with, to be more informative to a larger audience, we gave the party names in the tables and used their English translations.

Based on this data and exploiting the results of Section 3, we give under each table a complete account of strong monotonicity violations that entail logically possible monotonicity violations. Note that when a PW-upward violation is applicable, a PL-upward violation is not, vice versa, and we only consider the applicable one. When a monotonicity violation is logically possible, we determine the vote transfer scenarios that lead to that violation.

1965 French presidential election[9]

Party	Candidate	Round 1 vote	Round 2 vote
Union for the New Republic	**Charles de Gaulle**	44.65	**55.2**
Convention of Republican Institutions	François Mitterrand	31.72	44.8
Popular Republican Movement	Jean Lecanuet	15.57	
Tixier-Vignancour Comities	Jean-Louis Tixier-Vignancour	5.2	
European Liberal Party	Pierre Marcilhacy	1.71	
Miscellaneous left	Marcel Barbu	1.15	

Proposition 1965: For the 1965 French presidential election, neither a PW-upward violation nor a downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to de Gaulle, C_2 to Mitterrand, and C_3 to Lecanuet. As $50 - \#C_1 < \#C_2 - \#C_3$, by Proposition 1, a PW-upward(2) violation of strong monotonicity does not occur and by Proposition 3, a PW-upward(k) violation ($5 \geq k \geq 3$) of strong monotonicity does not occur either. Thus, a PW-upward violation of monotonicity is logically impossible. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

[9] The data is from:
https://www.conseil-constitutionnel.fr/decision/1965/656pdr.htm
https://www.conseil-constitutionnel.fr/decision/1965/6510pdr.htm

1969 French presidential election[10]

Party	Candidate	Round 1 vote	Round 1 vote
Union of Democrats for the Republic	**Georges Pompidou**	44.47	**58.21**
Democratic Center	Alain Poher	23.31	41.79
French Communist Party	Jacques Duclos	21.27	
French Section of the Workers' International	Gaston Defferre	5.01	
Unified Socialist Party	Michel Rocard	3.61	
Independent Radical Socialist	Louis Ducatel	1.27	
Communist League	Alain Krivine	1.06	

Proposition 1969: For the 1969 French presidential election, a PW-upward(k) violation of monotonicity is logically possible if and only if k=2 while no downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Pompidou, C_2 to Poher, C_3 to Duclos and C_4 to Defferre. As $50 - \#C_1 > \#C_2 - \#C_3$, by Proposition 1, a PW-upward(2) violation of strong monotonicity occurs, hence a PW-upward(2) violation of monotonicity is logically possible. As $50 - \#C_1 < (\#C_2 - \#C_3) + (\#C_2 - \#C_4)$, by Proposition 2, a PW-upward(3) violation of strong monotonicity does not occur and by Proposition 3, a PW-upward(k) violation ($6 \geq k \geq 4$) of strong monotonicity does not occur either. Thus, a PW-upward violation(k) of monotonicity is logically possible iff k=2. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

Scenario for PW-upward(2) violation: Pompidou gets x votes from Poher, $2.04 < x < 5.53$, which entails a runoff between Pompidou and Duclos, instead of Pompidou and Poher.

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[10] The data is from:
https://www.conseil-constitutionnel.fr/decision/1969/6920PDR.htm
https://www.conseil-constitutionnel.fr/decision/1969/6922pdr.htm

1974 French presidential election[11]

Party	Candidate	Round 1 vote	Round 2 vote
Socialist Party	**François Mitterrand**	43.25	49.19
Independent Republican	Valéry Giscard d'Estaing	32.6	**50.81**
Union of Democrats for the Republic	Jacques Chaban-Delmas	15.11	
Independent right-wing conservatives	Jean Royer	3.17	
Workers' Struggle	Arlette Laguiller	2.33	
Independent Environmentalist	René Dumont	1.32	
National Front	Jean-Marie Le Pen	0.75	
Democratic Socialist Movement of France	Émile Muller	0.69	
Revolutionary Communist front	Alain Krivine	0.37	
New Royalist Action	Bertrand Renouvin	0.17	
European Federalist Movement	Jean-Claude Sebag	0.16	
European federalist	Guy Héraud	0.08	

Proposition 1974: For the 1974 French presidential election, neither a PL-upward violation nor a downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Mitterrand, C_2 to Giscard d'Estaing, and C_3 to Chaban-Delmas. As $50 - \#C_2 < \#C_1 - \#C_3$, by Proposition 4, a PL-upward(1) violation of strong monotonicity does not occur and by Proposition 6, no PL-upward(k) violation of strong monotonicity occurs. Thus, a PL-upward violation of monotonicity is logically impossible. Since the plurality loser is the PwR winner, PW-upward violations do not occur. As $\#C_1 - \#C_2 < \#C_2 - \#C_3$, by Proposition 7, no downward(3) violation of strong monotonicity occurs, and by Corollary 1, no downward(k) ($k = 4, \dots, 12$) violation of strong monotonicity occurs either, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

[11] The data is from
https://www.conseil-constitutionnel.fr/decision/1974/7430pdr.htm
https://www.conseil-constitutionnel.fr/decision/1974/7432PDR.htm

1981 French presidential election[12]

Party	Candidate	Round 1 vote	Round 2 vote
Union for French Democracy	**Valéry Giscard d'Estaing**	28.32	48.24
Socialist Party	François Mitterrand	25.85	**51.76**
Rally for the Republic	Jacques Chirac	18	
French Communist Party	Georges Marchais	15.35	
Political Ecology Movement	Brice Lalonde	3.88	
Workers' Struggle	Arlette Laguiller	2.3	
Radical Party of the Left	Michel Crépeau	2.21	
Gaullist miscellaneous right	Michel Debré	1.66	
Gaullist miscellaneous right	Marie-France Garaud	1.33	
Unified Socialist Party	Huguette Bouchardeau	1.11	

Proposition 1981: For the 1981 French presidential election, a PL-upward(k) violation of monotonicity is logically possible if and only if k=1 or 3 while no downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Giscard d'Estaing, C_2 to Mitterrand, C_3 to Chirac and C_4 to Marchais. As $50 - \#C_2 > (\#C_1 - \#C_3)$, by Proposition 4, a PL-upward(1) violation of strong monotonicity occurs. Moreover, $50 - \#C_2 > (\#C_1 - \#C_4) + (\#C_3 - \#C_4)$, hence by Proposition 5, a PL-upward(3) violation of strong monotonicity occurs. Thus, for k=1 or 3, a PL-upward(k) violation of monotonicity is logically possible. Noting that C_5 corresponds to Lalonde, for k = 4, observe $50 - \#C_2 < (\#C_1 - \#C_5) + (\#C_3 - \#C_5) + (\#C_4 - \#C_5)$. Thus, by Proposition 5, a PL-upward(4) violation of strong monotonicity does not occur; and by Proposition 6, this applies to every k > 4, hence rendering a PL-upward(k) violation of monotonicity logically impossible when k exceeds 3.

As $\#C_1 - \#C_2 < \#C_2 - \#C_3$, by Proposition 7, no downward(3) violation of strong monotonicity occurs, and by Corollary 1, no downward(k) ($k = 4, \ldots, 10$) violation of strong monotonicity occurs either, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

[12] The data is from
https://www.conseil-constitutionnel.fr/decision/1981/8145pdr.htm
https://www.conseil-constitutionnel.fr/decision/1981/8147pdr.htm

Scenario for PL-upward(1) violation: Mitterrand gets x votes from Giscard d'Estaing, 10.32 < x < 24.15, which entails a runoff between Mitterrand and Chirac, instead of Mitterrand and Giscard d'Estaing.

Scenario for PL-upward(3) violation: Mitterrand gets $x > 12.97$ votes from Giscard d'Estaing, and $y > 2.65$ votes from Chirac, $x + y < 24.15$, which entails a runoff between Mitterrand and Marchais, instead of Mitterrand and Giscard d'Estaing.

1988 French presidential election[13]

Party	Candidate	Round 1 vote	Round 2 vote
Socialist Party	**François Mitterrand**	34.1	**54.02**
Rally for the Republic	Jacques Chirac	19.94	45.98
Union for French Democracy	Raymond Barre	16.55	
National Front	Jean-Marie Le Pen	14.39	
French Communist Party	André Lajoinie	6.76	
The Greens	Antoine Waechter	3.78	
Unified Socialist Party / Revolutionary Communist League	Pierre Juquin	2.1	
Workers' Struggle	Arlette Laguiller	1.99	
Movement for a Workers' Party	Pierre Boussel	0.38	

Proposition 1988: For the 1988 French presidential election, a PW-upward(k) violation of monotonicity is logically possible if and only if k=2 or 3 while no downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Mitterrand, C_2 to Chirac, C_3 to Barre, C_4 to Le Pen and C_5 to Lajoinie. Since $50 - \#C_1 > \#C_2 - \#C_3$, by Proposition 1, a PW-upward(2) violation of strong monotonicity occurs. In addition, as $50 - \#C_1 > (\#C_2 - \#C_3) + (\#C_2 - \#C_4)$, by Proposition 2, a PW-upward(3) violation of strong monotonicity occurs. Thus, for k=1 or 3, a PL-upward(k) violation of monotonicity is logically possible. Note that $50 - \#C_2 < (\#C_1 - \#C_5) + (\#C_3 - \#C_5) + (\#C_4 - \#C_5)$. Thus, by Proposition 2, a PW-

[13] The data is from
https://www.conseil-constitutionnel.fr/decision/1988/8856pdr.htm
https://www.conseil-constitutionnel.fr/decision/1988/8860PDR.htm

upward(4) violation of strong monotonicity does not occur; and by Proposition 3, this applies to every k > 4, hence rendering a PW-upward(k) violation of monotonicity logically impossible when k exceeds 3. Thus, a PW-upward violation(k) of monotonicity is logically possible iff k=2. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

Scenario for PW-upward(2) violation: Mitterrand gets x votes from Chirac, $3.39 < x < 19.94$,[14] which entails a runoff between Mitterrand and Barre, instead of Mitterrand and Chirac.

Scenario for PW-upward(3) violation: Mitterrand gets $x > 5.55$ votes from Chirac, and $y > 2.16$ votes from Barre, $x + y < 30.06$, which entails a runoff between Mitterrand and Le Pen instead of Mitterrand and Chirac.

1995 French presidential election[15]

Party	Candidate	Round 1 vote	Round 2 vote
Socialist Party	**Lionel Jospin**	23.3	47.36
Rally for the Republic	Jacques Chirac	20.84	**52.64**
RPR/ Union for French Democracy	Édouard Balladur	18.58	
National Front	Jean-Marie Le Pen	15	
French Communist Party	Robert Hue	8.64	
Workers' Struggle	Arlette Laguiller	5.3	
Movement for France	Philippe de Villiers	4.74	
The Greens	Dominique Voynet	3.32	
European Workers' Party	Jacques Cheminade	0.28	

Proposition 1995: For the 1995 French presidential election, a PL-upward(k) violation of monotonicity is logically possible if and only if k=1 or 3; and a downward(k) violation of monotonicity is logically possible if and only if k=3.

[14] Although the inequality in Proposition 1 requires $x < 30.06$, the actual vote of Chirac which is 19.94 is the binding upper bound on x. This issue applies to the 1995, 2002 and 2017 elections as well.

[15] The data is from
https://www.conseil-constitutionnel.fr/decision/1995/9581PDR.htm
https://www.france-politique.fr/election-presidentielle-1995.htm

Proof: In this election, C_1 corresponds to Jospin, C_2 to Chirac, C_3 to Balladur, C_4 to Le Pen and C_5 to Hue. As $50 - \#C_2 > (\#C_1 - \#C_3)$, by Proposition 4, a PL-upward(1) violation of strong monotonicity occurs. Moreover, $50 - \#C_2 > (\#C_1 - \#C_4) + (\#C_3 - \#C_4)$, hence by Proposition 5, a PL-upward(3) violation of strong monotonicity occurs. Thus, for k=1 or 3, a PL-upward(k) violation of monotonicity is logically possible. For k = 4, observe $50 - \#C_2 < (\#C_1 - \#C_5) + (\#C_3 - \#C_5) + (\#C_4 - \#C_5)$. Thus, by Proposition 5, a PL-upward(4) violation of strong monotonicity does not occur; and by Proposition 6, this applies to every k > 4, hence rendering a PL-upward(k) violation of monotonicity logically impossible when k exceeds 3.

As for downward violations, note that $\#C_1 - \#C_2 > \#C_2 - \#C_3$. Hence, by Proposition 7, downward(3) violation of strong monotonicity occurs; so a downward(3) violation of monotonicity is logically possible. Furthermore, we have $\#C_1 - \#C_2 < \#C_2 - \#C_4$. Therefore, by Proposition 7, downward(4) violation of strong monotonicity does not occur. By Corollary 1, downward(k) violation of strong monotonicity does not occur for any k > 4, hence rendering a downward(k) violation of monotonicity logically impossible when k exceeds 3. Q.E.D.

Scenario for PL-upward(1) violation: Chirac gets x votes from Jospin, $4.72 < x < 23.3$ (see footnote 14 for the upper bound on x), which entails a runoff between Chirac and Balladur, instead of Chirac and Jospin.

Scenario for PL-upward(3) violation: Chirac gets $x > 8.3$ votes from Jospin, and $y > 3.58$ votes from Balladur, $x + y < 29.16$, which entails a runoff between Chirac and Le Pen, instead of Chirac and Jospin.

Scenario for Downward(3) violation: Jospin loses x votes, $2.26 < x < 2.46$, to Balladur, which entails a runoff between Jospin and Balladur, instead of Chirac and Jospin.

2002 French presidential election[16]

Party	Candidate	Round 1 vote	Round 2 vote
Rally for the Republic	**Jacques Chirac**	19.88	**82.21**
National Front	Jean-Marie Le Pen	16.86	17.79
Socialist Party	Lionel Jospin	16.18	
Union for French Democracy	François Bayrou	6.84	
Workers' Struggle	Arlette Laguiller	5.72	
Citizens' Movement	Jean-Pierre Chevènement	5.33	
The Greens	Noël Mamère	5.25	
Revolutionary Communist League	Olivier Besancenot	4.25	
Hunting, Fishing, Nature, Traditions	Jean Saint-Josse	4.23	
Liberal Democracy	Alain Madelin	3.91	
French Communist Party	Robert Hue	3.37	
National Republican Movement	Bruno Mégret	2.34	
Radical Party of the Left	Christiane Taubira	2.32	
Citizenship, Action, Participation for the 21st Century	Corinne Lepage	1.88	
Forum of Social Republicans	Christine Boutin	1.19	
Workers' Party	Daniel Gluckstein	0.47	

Proposition 2002: For the 2002 French presidential election, a PW-upward(k) violation of monotonicity is logically possible if and only if $2 \leq k \leq 8$, while no downward violation of monotonicity is logically possible.

Proof: Column two of the table above gives the names of the candidates that we denote by C_1 through C_9 in descending order. Since $50 - \#C_1 > \sum_{i=2}^{k}(\#C_i - \#C_{k+1})$ for every $2 \leq k \leq$

[16] The data is from https://www.conseil-constitutionnel.fr/election-presidentielle-2002/bilan-du-premier-tour-de-l-election-presidentielle-de-2002 https://www.conseil-constitutionnel.fr/election-presidentielle-2002/bilan-du-second-tour-de-l-election-presidentielle-de-2002

8, by Proposition 1, a PW-upward(k) violation of strong monotonicity occurs for every $2 \leq k \leq 8$. Thus, for for every $2 \leq k \leq 8$, a PL-upward(k) violation of monotonicity is logically possible. Note that $50 - \#C_1 > \sum_{i=2}^{9}(\#C_i - \#C_{k+1})$. Thus, by Proposition 2, a PW-upward(9) violation of strong monotonicity does not occur; and by Proposition 3, this applies to every k > 9, hence rendering a PW-upward(k) violation of monotonicity logically impossible when k exceeds 8. Thus, a PW-upward violation(k) of monotonicity is logically possible iff $2 \leq k \leq 8$. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

Scenario for PW-upward(2) violation: Chirac gets *x* votes from Le Pen, 0.68 < *x* < 16.86 (see footnote 14 for the upper bound on *x*), which entails a runoff between Chirac and Jospin instead of Chirac and Le Pen.

Scenario for PW-upward(3) violation: Chirac gets *x* > 10.02 votes from Le Pen, and *y* > 9.34 votes from Jospin, *x* + *y* < 30.12, which entails a runoff between Chirac and Bayrou instead of Chirac and Le Pen.

Scenario for PW-upward(4) violation: Chirac gets x_2 > 11.14 votes from Le Pen, x_3 > 10.46 votes from Jospin, x_4 > 1.12 votes from Bayrou, x_2 + x_3 + x_4 < 30.12, which entails a runoff between Chirac and Laguiller instead of Chirac and Le Pen.

Scenario for PW-upward(5) violation: Chirac gets x_2 > 11.53 votes from Le Pen, x_3 > 10.85 votes from Jospin, x_4> 1.51 votes from Bayrou, x_5 > 0.39 votes from Laguiller, x_2 + x_3 + x_4 + x_5 < 30.12, which entails a runoff between Chirac and Chevènement instead of Chirac and Le Pen.

Scenario for PW-upward(6) violation: Chirac gets x_2 > 11.61 votes from Le Pen, x_3 > 10.93 votes from Jospin, x_4 > 1.59 votes from Bayrou, x_5 > 0.47 votes from Laguiller, x_6 > 0.08 votes from Chevènement, x_2 + x_3 + x_4 + x_5 + x_6 < 30.12, which entails a runoff between Chirac and Mamère instead of Chirac and Le Pen.

Scenario for PW-upward(7) violation: Chirac gets x_2 > 12.61 votes from Le Pen, x_3 > 11.93 votes from Jospin, x_4 > 2.59 votes from Bayrou, x_5 > 1.47 votes from Laguiller, x_6 > 1.08 votes from Chevènement, x_7 > 1 votes from Mamère, x_2 + x_3 + x_4 + x_5 + x_6 + x_7 < 30.12, which entails a runoff between Chirac and Besancenot instead of Chirac and Le Pen.

Scenario for PW-upward(8) violation: Chirac gets x_2 > 12.63 votes from Le Pen, x_3 > 11.95 votes from Jospin, x_4 > 2.61 votes from Bayrou, x_5 > 1.49 votes from Laguiller, x_6 > 1.1 votes from Chevènement, x_7 > 1.02 votes from Mamère, x_8 > 0.02 votes from Besancenot, x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 < 30.12, which entails a runoff between Chirac and Saint-Josse instead of Chirac and Le Pen.

2007 French presidential election[17]

Party	Candidate	Round 1 vote	Round 2 vote
Union for a Popular Movement	**Nicolas Sarkozy**	31.18	**53.06**
Socialist Party	Ségolène Royal	25.87	46.94
Union for French Democracy	François Bayrou	18.57	
National Front	Jean-Marie Le Pen	10.44	
Revolutionary Communist League	Olivier Besancenot	4.08	
Movement for France	Philippe de Villiers	2.23	
French Communist Party	Marie-George Buffet	1.93	
The Greens	Dominique Voynet	1.57	
Workers' Struggle	Arlette Laguiller	1.33	
Miscellaneous left	José Bové	1.32	
Hunting, Fishing, Nature, Traditions	Frédéric Nihous	1.15	
Workers' Party	Gérard Schivardi	0.34	

Proposition 2007: For the 2007 French presidential election, a PW-upward(k) violation of monotonicity is logically possible if and only if k=2 while no downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Sarkozy, C_2 to Royal, C_3 to Bayrou and C_4 to Le Pen. As $50 - \#C_1 > \#C_2 - \#C_3$, by Proposition 1, a PW-upward(2) violation of strong monotonicity occurs, hence a PW-upward(2) violation of monotonicity is logically possible. As $50 - \#C_1 < (\#C_2 - \#C_3) + (\#C_2 - \#C_4)$, by Proposition 2, a PW-upward(3) violation of strong monotonicity does not occur and by Proposition 3, a PW-upward(k) violation ($11 \geq k \geq 4$) of strong monotonicity does not occur either. Thus, a PW-upward violation(k) of monotonicity is logically possible iff k=2. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

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[17] The data is from
https://www.conseil-constitutionnel.fr/election-presidentielle-2007/bilan-du-premier-tour
https://www.conseil-constitutionnel.fr/decision/2007/2007141pdr.htm

Scenario for PW-upward(2) violation: Sarkozy gets x votes from Royal, $7.3 < x < 18.82$ which entails a runoff between Sarkozy and Bayrou instead of Sarkozy and Royal.

2012 French presidential election[18]

Party	Candidate	Round 1 vote	Round 1 vote
Socialist Party	**François Hollande**	28.63	**51.64**
Union for a Popular Movement	Nicolas Sarkozy	27.18	48.36
National Front	Marine Le Pen	17.9	
Left Front	Jean-Luc Mélenchon	11.1	
Democratic Movement	François Bayrou	9.13	
Europe Ecology/ The Greens	Eva Joly	2.31	
France Arise	Nicolas Dupont-Aignan	1.79	
New Anticapitalist Party	Philippe Poutou	1.15	
Workers' Struggle	Nathalie Arthaud	0.56	
Solidarity and Progress	Jacques Cheminade	0.25	

Proposition 2012: For the 2012 French presidential election, a PW-upward(k) violation of monotonicity is logically possible if and only if k=2 while no downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Hollande, C_2 to Sarkozy, C_3 to Le Pen and C_4 to Mélenchon. As $50 - \#C_1 > \#C_2 - \#C_3$, by Proposition 1, a PW-upward(2) violation of strong monotonicity occurs, hence a PW-upward(2) violation of monotonicity is logically possible. As $50 - \#C_1 < (\#C_2 - \#C_3) + (\#C_2 - \#C_4)$, by Proposition 2, a PW-upward(3) violation of strong monotonicity does not occur and by Proposition 3, a PW-upward(k) violation ($9 \geq k \geq 4$) of strong monotonicity does not occur either. Thus, a PW-upward violation(k) of monotonicity is logically possible iff k=2. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

[18] The data is from
https://www.conseil-constitutionnel.fr/decision/2012/2012152PDR.htm
https://www.conseil-constitutionnel.fr/decision/2012/2012154PDR.htm

Scenario for PW-upward(2) violation: Hollande gets *x* votes from Sarkozy, 9.28 < *x* < 21.37 which entails a runoff between Hollande and Le Pen instead of Hollande and Sarkozy.

2017 French presidential election[19]

Party	Candidate	Round 1 vote	Round 2 vote
The Republic Forward	**Emmanuel Macron**	24.01	**66.1**
National Front	Marine Le Pen	21.3	33.9
The Republicans	François Fillon	20.01	
Unbowed France	Jean-Luc Mélenchon	19.58	
Socialist Party	Benoit Hamon	6.36	
France Arise	Nicolas Dupont-Aignan	4.7	
Resistions	Jean Lassalle	1.21	
New Anticapitalist Party	Philippe Poutou	1.09	
Popular Republican Movement	François Asselineau	0.92	
Workers' Struggle	Nathalie Arthaud	0.64	
Solidarity and Progress	Jacques Cheminade	0.18	

Proposition 2017: For the 2017 French presidential election, a PW-upward(k) violation of monotonicity is logically possible if and only if k=2 or 3 while no downward violation of monotonicity is logically possible.

Proof: In this election, C_1 corresponds to Macron, C_2 to Le Pen, C_3 to Fillon, C_4 to Mélenchon and C_5 to Hamon. Since $50 - \#C_1 > \#C_2 - \#C_3$, by Proposition 1, a PW-upward(2) violation of strong monotonicity occurs. In addition, as $50 - \#C_1 > (\#C_2 - \#C_3) + (\#C_2 - \#C_4)$, by Proposition 2, a PW-upward(3) violation of strong monotonicity occurs. Thus, for k=1 or 3, a PL-upward(k) violation of monotonicity is logically possible. Note that, $50 - \#C_2 < (\#C_1 - \#C_5) + (\#C_3 - \#C_5) + (\#C_4 - \#C_5)$. Thus, by Proposition 2, a PW-upward(4) violation of strong monotonicity does not occur; and by Proposition 3, this applies to every k > 4, hence rendering a PW-upward(k) violation of monotonicity logically impossible when k

[19] The data is from
https://www.conseil-constitutionnel.fr/decision/2017/2017169PDR.htm
https://www.conseil-constitutionnel.fr/decision/2017/2017171PDR.htm

exceeds 3. Thus, a PW-upward violation(k) of monotonicity is logically possible iff k=2. As the plurality loser is not the PwR winner, by Proposition 7, no downward violation of strong monotonicity occurs, hence rendering a downward violation of monotonicity logically impossible. Q.E.D.

Scenario for PW-upward(2) violation: Macron gets x votes from Le Pen, $1.29 < x < 21.3$ (see footnote 14 for the upper bound on x), which entails a runoff between Macron and Fillon instead of Macron and Le Pen.

Scenario for PW-upward(3) violation: Macron gets $x > 1.72$ votes from Le Pen, and $y > 0.43$ votes from Fillon, $x + y < 25.99$, which entails a runoff between Macron and Mélenchon instead of Macron and Le Pen.

Table 1 below summarizes our findings regarding the 10 French presidential elections. It gives, for each election, all PW-upward(k), PL-upward(k) or downward(k) violations that are possible, together with the required vote shift (when k=2 for PW-upward, k=1 for PL-upward, and k=3 for downward violations), as well as the change in the runoff pair under the violation scenario in columns 6 and 9. The format we adopted in those two columns reads as: the actual pair that raced in runoff > the new pair in runoff after the vote shifts in columns 5 and 8, respectively.

Table 1

Election	Winner	Maximal k for PW-upward(k) violation	Maximal k for PL-upward(k) violation	Min vote shift required for PW-upward(2) or PL-upward(1)	Change in runoff pair (PW-upward(2) or PL-upward(1))	Maximal k for downward(k) violation	Min vote shift required for downward(3)	Change in runoff pair (downward(3))
2017	Macron	3	N.A.	1.29 (Le Pen → Macron)	Macron & Le Pen > Macron & Fillon	N.A.	N.A.	N.A.
2012	Hollande	2	N.A.	9.28 (Sarkozy → Hollande)	Hollande & Sarkozy > Hollande & Le Pen	N.A.	N.A.	N.A.
2007	Sarkozy	2	N.A.	7.3 (Royal → Sarkozy)	Sarkozy & Royal > Sarkozy & Bayrou	N.A.	N.A.	N.A.
2002	Chirac	8	N.A.	0.68 (Le Pen → Chirac)	Chirac & Le Pen > Chirac & Jospin	N.A.	N.A.	N.A.
1995	Chirac	N.A.	3	4.72 (Jospin → Chirac)	Chirac & Jospin > Chirac & Balladur	3	2.26 (Jospin → Balladur)	Jospin & Chirac > Jospin & Balladur
1988	Mitterrand	3	N.A.	3.39 (Chirac → Mitterrand)	Mitterand & Chirac > Mitterand & Barre	N.A.	N.A.	N.A.
1981	Mitterrand	N.A.	3	10.32 (d'Estaing → Mitterrand)	Mitterand & d'Estaign > Mitterand & Chirac	NONE	N.A.	N.A.
1974	d'Estaing	N.A.	NONE	N.A	N.A.	NONE	N.A.	N.A.
1969	Pompidou	2	N.A.	2.04 (Poher → Pompidou)	Pompidou & Poher > Pompidou & Duclos	N.A.	N.A.	N.A.
1965	de Gaulle	NONE	N.A.	N.A	N.A.	N.A.	N.A.	N.A.

Table 1: Summary of strong monotonicity violations.

5. French presidentials: Practically possible monotonicity violations

Every scenario of strong monotonicity violation consists of a vote transfer from one candidate to another, accompanied by a change in the pair of candidates that go for a runoff. Whether the violation of strong monotonicity leads to a monotonicity violation that is practically possible depends on two factors:

- Does the change in the runoff pair would effectively result in a change in the winning candidate?
- Does the related vote transfer is plausible within the considered political environment?

We address these two issues in the sequel.

5.1 Change in the runoff pair versus change in the winning candidate

We start by discussing whether the changes in the runoff pairs, expressed in columns 6 and 9 of Table 1, could effectively change the winning candidates. For each election where a strong monotonicity violation is logically possible, we qualify the occurrence of a change in the winning candidate by one of the following three labels: LIKELY, PROBABLE, UNLIKELY.[20]

For the 1969 election, through the detailed analysis of Penniman (1969) on the matter, it can be safely claimed that Pompidou would win against Duclos. Hence, we conclude that the occurrence of a change is UNLIKELY.

For the 1981 election, PL-upward(1) and PL-upward(3) violations were possible. For PL-upward(1), total vote of left is above 50% in the first round. Moreover, as the left was more united compared to the right and the loss of right was mainly attributed to the vote shift from right to left over the years of d'Estaing rule[21] (Criddle & Bell (1981)), Mitterrand would probably win against Chirac. We conclude that the occurrence of a change is UNLIKELY. PL-upward(3) violation involves a runoff between Mitterrand and the communist candidate Marchais. As discussed in Criddle & Bell (1981), there were more communists who would support a socialist candidate than there were socialists who would support a communist candidate; and the right preferred a socialist candidate rather than a communist one. As a result of these arguments, we conclude that a change is UNLIKELY.

For the 1988 election, PW-upward(2) and PW-upward(3) violations were possible. For PW-upward(2), polls about a Barre - Mitterrand runoff generally indicate Mitterrand as the winner[22],

[20] Some changes may be more unlikely than others but for the purpose of our analysis a single category that reflects unlikeliness suffices.

[21] Among the reasons for such a shift was the steady move of Christian west towards left and the popularity of socialism among youngsters who vote for the first time in 1981.

[22] https://www.ipsos.com/fr-fr/barometre-election-presidentiellevote-transfert-de-voix-cote-des-candidats
https://www.ipsos.com/fr-fr/barometre-cote-des-presidentiables-et-intentions-de-vote
https://www.ipsos.com/fr-fr/presidentielle-88-intentions-de-vote-et-candidature-le-pen
https://www.ipsos.com/fr-fr/barometre-election-presidentiellevote-transfert-de-voix-cote-des-candidats-0

an indication that particularly prevails when the election date gets closer.[23] We conclude that the occurrence of a change is UNLIKELY. As for PW-upward(3), it can be claimed that Mitterrand would win against Le Pen (see Frears (1988) for a detailed analysis). Furthermore, taking into account the second round of the 2002 election where Le Pen lost against Chirac with an overwhelming majority, we can safely claim that Le Pen would lose against Mitterrand in 1988, hence concluding that the occurrence of a change is UNLIKELY.

The 1995 election is open to PL-upward(1), PL-upward(3) and downward(3) violations. For the first one, as the elections date approaches, polls about a Chirac-Balladur runoff indicate Chirac as the winner[24] and we conclude that the occurrence of a change is UNLIKELY. PL-upward(3) violation scenario involves a runoff between Chirac and Le Pen. This race actually took place in 2002 elections in which Chirac won by an overwhelming majority of votes against Le Pen. Therefore, we can easily conclude that a change for PL-upward(3) in 1995 is UNLIKELY. Downward(3) violation of monotonicity requires Jospin win against Balladur, which seems UNLIKELY according to the polls.[25]

For the 2002 election, regarding the PW-upward(2) violation, polls were indecisive between Chirac and Jospin.[26] We conclude that the occurrence of a change is PROBABLE.[27]

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[23] https://www.ipsos.com/fr-fr/barometre-cote-des-presidentiables-et-intentions-de-vote

[24] https://www.liberation.fr/evenement/1995/03/03/sondages-chirac-devance-balladur-au-second-tour_128284
https://www.liberation.fr/france-archive/1995/03/08/sondages-chirac-donne-vainqueur-dans-tous-les-cas-de-figure_127767
https://www.liberation.fr/france-archive/1995/03/24/sondages-chirac-recule-mais-gagne-dans-tous-les-cas-de-figure_126037
https://www.liberation.fr/france-archive/1995/03/29/sondages-chirac-fait-du-sur-place-balladur-en-profite-un-peu_125598
https://www.liberation.fr/france-archive/1995/03/30/sondage-derriere-chirac-jospin-devance-balladur-de-trois-points_125427
https://www.ipsos.com/fr-fr/intentions-de-vote-la-presidentielle-21

[25] https://www.liberation.fr/evenement/1995/03/03/sondages-chirac-devance-balladur-au-second-tour_128284
https://www.liberation.fr/france-archive/1995/03/08/sondages-chirac-donne-vainqueur-dans-tous-les-cas-de-figure_127767
https://www.liberation.fr/france-archive/1995/03/15/chirac-reste-en-tete-selon-un-sondage_127015
https://www.liberation.fr/france-archive/1995/03/29/sondages-chirac-fait-du-sur-place-balladur-en-profite-un-peu_125598
https://www.liberation.fr/france-archive/1995/03/30/sondage-derriere-chirac-jospin-devance-balladur-de-trois-points_125427
https://www.ipsos.com/fr-fr/intentions-de-vote-la-presidentielle-21

[26] https://web.archive.org/web/20020425110704/http://www.ifop.com/europe/sondages/OPINIONF/presidv8.asp
https://web.archive.org/web/20061123034026/http://www.lh2.fr/_upload/ressources/sondages/election/LHFLibeAOLIntentdevote1617avril.pdf
https://web.archive.org/web/20030822222733/http://www.tns-sofres.com/etudes/pol/190402_intentions_r.htm
https://www.ipsos.com/fr-fr/le-barometre-presidentielle-legislatives-2002-vague-7
https://web.archive.org/web/20020618193356/http://www.csa-fr.com/fra/dataset/data2002/indic20020418.htm
https://web.archive.org/web/20020611091604/http://www.bva.fr/new/baro_observatoire_presidentiel20011127.html

[27] At the end of our analysis, we are able to qualify the 2002 election as one which is practically open to an upward 1.1 monotonicity violation .Thus, we do not analyze the plausibility of other 1.k violation scenarios for k > 1.

For the 2007 election, polls about a Bayrou - Sarkozy runoff generally indicate Bayrou as the winner, an indication that particularly prevails when the election date gets closer.[28] We conclude that the occurrence of a change is LIKELY.

For the 2012 election, polls about a Hollande-Le Pen runoff very clearly indicate Hollande as the winner.[29] Thus, we conclude that the occurrence of a change is UNLIKELY.

The 2017 election is vulnerable to both PW-upward(2) and PW-upward(3) violations. For PW-upward(2), polls about a Fillon-Macron runoff clearly indicate Macron as the winner, an indication that gets stronger when the election date gets closer.[30] We conclude that the occurrence of a change is UNLIKELY. Regarding the PW-upward(3) violation, polls about a Mélenchon-Macron runoff clearly indicate Macron as the winner.[31] We conclude that the occurrence of a change is UNLIKELY.

Overall, there are only two elections, namely those held in 2002 and 2007, where we think that the change in the runoff pair could effectively change the winning candidate. Recalling the vote transfers needed to change the runoff pair in these two elections, we can conclude that

- in the 2002 election, a transfer of 0.68 percent from Le Pen to Chirac would lead to an upward monotonicity violation that is PROBABLE, making Chirac lose an election that he had win;
- in the 2007 election, a transfer of 7.3 percent from Royal to Sarkozy would lead to an upward monotonicity violation that is LIKELY, making Sarkozy lose an election that he had win.

5.2 Plausibility of the vote transfer

[28] https://www.ifop.com/wp-content/uploads/2018/03/intentions2203.pdf
https://web.archive.org/web/20070913134736/http://www.lh2.fr/_upload/ressources/sondages/politique_nationale/lh2rmcbfmtv20mn26_27mars07.pdf
https://www.ifop.com/wp-content/uploads/2018/03/flash_presi.pdf
https://web.archive.org/web/20070605010944/http://www.ipsos.fr/presidentielle-2007/pdf/200407-2.pdf

[29] https://web.archive.org/web/20110516022607/http://www.lh2.fr/_upload/ressources/sondages/politique_nationale/lh2yahoointentionsvotepresidentielle2012_07mai11.pdf
https://www.ifop.com/wp-content/uploads/2018/03/1480-1-study_file.pdf

[30] psos.com/sites/default/files/files-fr-fr/doc_associe/rapport_vague13.pdf
http://harris-interactive.fr/wp-content/uploads/sites/6/2017/04/rapport-Harris-Indeed-Intentions-vote-election-presidentielle-LCP.pdf
https://elabe.fr/wp-content/uploads/2017/04/20042017_bfmtv_lexpress_intentions-de-vote-presidentielles-vague-10.pdf
http://www.odoxa.fr/wp-content/uploads/2017/04/Intention-de-vote-presidentielle-Odoxa-LePoint_210417.pdf

[31] https://www.ipsos.com/sites/default/files/files-fr-fr/doc_associe/rapport_vague13.pdf
http://harris-interactive.fr/wp-content/uploads/sites/6/2017/04/rapport-Harris-Indeed-Intentions-vote-election-presidentielle-LCP.pdf
https://elabe.fr/wp-content/uploads/2017/04/20042017_bfmtv_lexpress_intentions-de-vote-presidentielles-vague-10.pdf

We now discuss whether the vote transfers expressed in column 5 and 8 of Table 1, support the plausibility of the monotonicity violation scenarios for the 2002 and 2007 elections. To this end, we elaborate on the mechanisms that can lead to vote transfers as well as the ideological positioning of the candidates.[32]

5.2.1 Mechanisms for vote transfers

We start by a conceptual distinction. Suppose some votes of candidate C are transferred to candidate $C^{'}$. When this transfer is a result of a campaign made by $C^{'}$, we qualify it as natural. However, if it is C who asks his supporters to vote for $C^{'}$, then we qualify it as manipulative. To see the point of this distinction, consider a downward manipulation where C_1 and C_2 (with $\#C_1 > \#C_2$) go for a runoff where C_2 wins. The condition $\#C_1 - \#C_2 > \#C_2 - \#C_3$ expressed by Proposition 7 is satisfied. Thus, a transfer of x votes with $\#C_1 - \#C_2 > x > \#C_2 - \#C_3$ from C_1 to C_3 allows C_1 and C_3 to go to the runoff and let, for the sake of argument, C_1 be the winner. Now, the source of this vote transfer can be a successful propaganda made by C_3 (hence a natural transfer) but it can be very well the case that C_1 who anticipates this situation may coordinate some of his supporters to vote for C_3 (hence a manipulative transfer).[33] While downward manipulations may admit both natural and manipulative vote transfers, the latter does not make sense for upward manipulations where the vote loser can never become the winner.

Note that, every vote transfer is expressed by two elements: an ordered pair of candidates and a quantity of votes. For natural vote transfers, it is the former that is critical because once a vote transfer from candidate C_2 to candidate C_1 is deemed to be possible, the monotonicity violation entails perverse initiatives over the campaign of C_1, independent of the associated quantity of votes. For manipulative vote transfers, by the very definition of the concept, a transfer between any two candidates is possible but here it is the reasonability of associated quantity of votes that must be considered.

The monotonicity violations in 2002 and 2007 are upward, hence are only open to natural vote transfers.

5.2.2 Ideological positioning of candidates

As the vote transfers in the 2002 and 2007 elections are natural, the ideological positioning of the candidates matters in assessing their plausibility. Considering the poll estimates before the election and the political positions of the two candidates, a natural vote transfer from Le Pen to Chirac in 2002 is reasonable (see Durand et al. (2004) for details). In the 2007 election, Sarkozy focused his campaign on popular classes and left aside the more educated class (see Sauger 2007). A more inclusive campaign would make a natural vote transfer from Royal to Sarkozy

[32] We thank an anonymous referee for suggesting this perspective.

[33] A well-known instance of manipulative vote transfers under non-monotonic voting rules is the referendum in Italy held on 12 June 2005 under the (non-monotonic) majority with quorum rule, where the Catholic Church, being against the withdrawal of a law dealing with medically assisted procreation, asked its supporters to abstain rather than casting a negative vote (Houy (2009)).

possible. Thus, we conclude that in both elections, the presumed vote transfers support the plausibility of the related monotonicity violation scenarios.

6. FINAL REMARKS

Identifying conditions that characterize voting situations where PwR is logically open to a monotonicity violation, we analyze all French presidential elections held under PwR from this perspective. Our analysis covers ten elections, from 1965 to 2017, and we show that eight of them (except the 1965 and 1974 elections) were logically open to a monotonicity violation. We also consider the circumstances specific to each of these eight elections and argue that two of them (namely the 2002 and 2007 elections) were practically vulnerable to a monotonicity violation. The following table summarizes our conclusions.

Table 2

Election	A monotonicity violation is logically	Changing the winning candidate under the motononicity violation scenario is	Do the presumed vote transfers support the monotonicity violation scenario?
2017	Possible	Unlikely	Not considered
2012	Possible	Unlikely	Not considered
2007	Possible	Likely	Yes
2002	Possible	Probable	Yes
1995	Possible	Unlikely	Not considered
1988	Possible	Unlikely	Not considered
1981	Possible	Unlikely	Not considered
1974	Impossible	Not Applicable	Not Applicable
1969	Possible	Unlikely	Not considered
1965	Impossible	Not Applicable	Not Applicable

Table 2: Summary of monotonicity violations.

Monotonicity violations appear to be more of a relatively recent issue. In fact, none of the six elections between 1965 and 1995 is vulnerable to a monotonicity violation while two of the four elections between 2002 and 2017 are so.

The theoretical computations of Lepelley et al. (1996)) suggest that, when the number of voters grows arbitrarily, the ratio of the number of voting situations with a monotonicity violation to the number of logically conceivable voting situations is 0.06482 (see Table 1 in page 140 of Lepelley et al. (1996)). Our observations reflect a higher vulnerability. A reason for this divergence may be that Lepelley et al. (1996)) consider elections with three candidates while in French presidential elections the number of candidates is more than three. In fact, Quas (2004) proposes a model where the frequency of monotonicity violations under IRV increases when the number of candidates increases. This observation does not directly apply to PwR which diverges from IRV when there are more than three candidates but it forms a basis to think that a similar incidence would hold for PwR as well.

We find it useful to contrast our findings with other papers that suggest lower frequency of monotonicity failures than our analysis. Graham - Squire and Zayatz (2020) studied 135 mayoral elections in two states of the USA run under IRV to see if they were vulnerable to monotonicity failures. These elections can be considered as PwR elections, as most of them were restricted to three candidates. On the other hand, 77% of them concluded in the first round, a level of agreement that never happened in French presidentials. Thus, we attribute the contrasting findings of our paper and Graham - Squire and Zayatz (2020) to the significant difference between the considered political environments. A similar claim is made by Bradley (1995) for elections in Ireland, but as the analysis leading to this conclusion is not presented, we are unable to comment on this claim.

The picture drawn by our analysis is more compatible with the findings of Miller (2017, Table 1, page 100) whose simulated data for three candidates suggests high frequency results. In a similar vein, the three-candidate spatial voting model of Ornstein and Norman (2014) estimates a lower bound of 15% for the frequency of monotonicity failures. Both papers consider IRV, which is equivalent to PwR when there are three candidates. Our results comply with these findings and indicate that the non-monotonicity of PwR should be a concern, at least within the French political context.

We close our analysis by making a remark on manipulations in French presidential elections. As Muller and Satterthwaite (1977) show, a social choice function being non-manipulable is equivalent to a stronger monotonicity condition than the one we analyze in this paper. Therefore, those social choice functions that violate our weaker monotonicity are prone to manipulation. Van der Straeten et al. (2013) designed an online experiment for the 2012 French presidential election where they observe that 13% of the participants do not vote for their most preferred candidate.

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